KIAS-P98015

The Power Of Effective Field Theories In Nuclei: The Deuteron, NN Scattering and Electroweak Processes

Tae-Sun Park[#1]

Grupo de Física Nuclear, Universidad de Salamanca, 37008 Salamanca, Spain

and

School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea

Kuniharu Kubodera[#2]

Department of Physics and Astronomy, University of South Carolina

Columbia, SC 29208, U.S.A.

Dong-Pil Min[#3]

Department of Physics and Center for Theoretical Physics

Seoul National University, Seoul 151-742, Korea

Mannque Rho[#4]

Service de Physique Théorique, CEA Saclay

91191 Gif-sur-Yvette Cedex, France

Abstract

We show how *effectively* effective quantum field theories work in nuclear physics. Using the physically transparent cut-off regularization, we study the simplest nuclear systems of two nucleons for both bound and scattering states at a momentum scale much less than the pion mass. We consider all the static properties of the deuteron, the two-nucleon scattering phase-shifts, the $n+p \to d+\gamma$ process at thermal energy and the solar proton fusion process $p+p \to d+e^+ +\nu_e$, and we demonstrate that these are all described with great accuracy in the expansion to the next-to-leading order. We explore how a "new" degree of freedom enters in an effective theory by turning on and off the role of the pion in the Lagrangian.

PACS: 21.30.Fe, 03.65.Nk, 13.75.Cs, 25.40.Lw, 26.65.+t

Key words: effective field theory, NN scattering, deuteron, radiative proton capture, solar proton fusion

[#1] Corresponding auther. e-mail: park@alph02.triumf.ca, tel.: 1-604-222-1047, fax: 1-604-222-1074, postal address: TRIUMF, Wesbrook Mall, Vancouver, B. C., V6T 2A3, Canada

[#2] e-mail: kubodera@nuc003.psc.sc.edu

[#3] e-mail: dpmin@phya.snu.ac.kr

[#4] e-mail: rho@spht.saclay.cea.fr

1 Introduction

The idea of effective quantum field theory[1, 2, 3] is extremely simple. At low energy where non-perturbative effects of QCD dominate, the relevant degrees of freedom are not quarks and gluons but hadrons. Let the relevant hadronic degrees of freedom be represented by the generic field Φ. Separate the field into two parts; the low-energy part Φ_L in which we are interested and the high-energy part Φ_H which does not interest us directly. We delineate the two parts at the momentum scale characterized by a cutoff, say, Λ_1. In the generating functional Z that we want to compute, integrate out the Φ_H field and define an effective action $S^{eff}_{\Lambda_1}[\Phi_L] = \frac{1}{i}\ln\left(\int[d\Phi_H]e^{iS[\Phi_L,\Phi_H]}\right)$. Then what we need to compute is

$$Z = \int_{\Lambda_1}[d\Phi_L]e^{iS^{eff}_{\Lambda_1}[\Phi_L]}. \tag{1}$$

Given the right degrees of freedom effective below the cutoff Λ_1, we expand the effective action as $S^{eff}_{\Lambda_1}[\Phi_L] = \sum_i^\infty C_i Q_i[\Phi_L]$, where $Q_i[\Phi_L]$ are local field operators allowed by the symmetries of the problem, and C_i's are constants, which are required to satisfy the "naturalness condition". The $Q_i[\Phi_L]$'s are ordered in such a manner that, for low-energy processes, the importance of the $Q_i[\Phi_L]$ term diminishes as i increases. The contribution of the Φ_H degrees of freedom integrated out from the action is not simply discarded but instead gets lodged in the coefficients C_i as well as in higher-dimensional operators. The strategy of effective field theories is to truncate the series at a manageable finite order, and obtain desired accuracy to capture the essence of the physics involved. The separation of Φ into Φ_L and Φ_H involves two steps: The first is to "decimate" the degrees of freedom, that is, to choose the fields Φ_L relevant for the physics to be incorporated explicitly into the Lagrangian, and the second is a regularization procedure which regulates the high-momentum contribution. Let m be the lightest mass of the degrees of freedom that are integrated out. Clearly the most physically transparent regularization is to introduce a momentum cutoff with $\Lambda_1 \sim m$. A different cutoff introduces different coefficients C_i for the given set of fields. The idea is then to pick a value of the cutoff that is low enough to avoid unnecessary complications but high enough to avoid throwing away relevant physics.

In the full (un-truncated) theory, the location of the cutoff is entirely arbitrary, so we could have chosen any values of the cutoff other than Λ_1 without changing physics. The cut-off dependence of $Q_i[\Phi_L]$ is compensated by the corresponding changes in C_i through the renormalization group equation. In such a theory, which is rarely available in four dimensions, physical observables should be strictly independent of where the cutoff is set. This of course will not be the case when the series is truncated. However, for a suitably truncated effective theory to be predictive, the observables should be more or less insensitive to where the cutoff is put. Should there be strong cutoff dependence, it would signal that there is something amiss in the scheme. For instance, it could be a signal for the presence of new degrees of freedom or of "new physics" that must be incorporated and/or for the necessity of including higher-order terms. How "new physics" enters in our case will be

studied in terms of the role that the pion – a "new degree of freedom" – plays at very low energy. Here we of course know exactly what the "new physics" is but there will be a lesson to learn from this exercise in exploring beyond "standard models". The following "toy model" illustrates our point. Let us imagine that we knew that QCD is the correct microscopic theory for the strong interactions with chiral symmetry realized in the Nambu-Goldstone mode but the pions were not yet discovered and that the only "effective" degrees of freedom we had at hand were the proton and the neutron. Now the question to ask is how to set up a systematic theory that could describe very low-energy properties of the deuteron, the NN scattering and the electroweak matrix elements and in doing so how to infer that a "new degree of freedom" (pion) lurked at ~ 140 MeV. We will see in Section 6 how this question could be studied in the real low-energy world of nuclei.

An important aspect of effective field theory is that the result should not depend upon what sort of regularization one uses. In principle, therefore, one could equally well use the dimensional regularization or the cut-off regularization or something else as long as one makes sure that all relevant symmetries are properly implemented [#1]. Calculated to high enough order in a consistent expansion, the results should in principle be the same up to that order although in practice there is a certain amount of art involved in the choice of the scheme. This issue in the context of the dimensional regularization and its variants was addressed recently in [4]. Here we will use the cut-off regularization for the reason that the procedure is simple and physically transparent [5, 6] when limited to the order we shall adopt, namely, to the next-to-leading order (NLO). For some of the problems treated here such as neutron-proton (np) scattering, one could also use in a predictive manner a modified dimensional regularization in which certain power divergences are removed (i.e., PDS scheme)[7]. For further comments on the PDS scheme, see Gegelia [4]. It is not yet fully clear to us how such a scheme despite its advantages could avoid some of the problems raised in [4, 6, 5].

In this paper, we show that an effective field theory with finite cut-off regularization works surprisingly well for all low-energy observables in the two-nucleon bound and scattering states; e.g., the static properties of the deuteron, the low-energy np scattering, the radiative np capture, $n + p \to d + \gamma$, at thermal energy and the proton fusion, $p + p \to d + e^+ + \nu_e$, in the Sun. Some of the results were reported in previous publications [8, 9, 10] but they will be included here both for the sake of completeness and for presenting our more recent sharpened thoughts on the matter. One of the significant results of this paper is a quantitative justification from "first principles" of the previously formulated "hybrid method" [8] for the radiative np capture process $n + p \to d + \gamma$. We justify quantitatively the procedure of using the high-quality phenomenological two-nucleon

[#1] There are of course certain issues which raise technical, though not fundamental, problems, e.g., how to preserve chiral symmetry in the cut-off procedure and how to correctly handle short-distance physics in the standard dimensional regularization etc. For a recent discussion on this matter, see [6].

wavefunctions[#2] supplied by the Argonne v_{18} potential [11] in calculating the matrix element of the electroweak operators derived by chiral perturbation theory with "irreducible" diagrams, a procedure first suggested by Weinberg [12] (for an early implementation, see [13]). Similar support is given for the hybrid-method calculation based on the high-quality phenomenological wavefunctions of the proton fusion process $p + p \to d + e^+ + \nu_e$ crucial in the solar neutrino problem. This constitutes an important application of effective theories for nuclei to astrophysics.

2 Formulation of the Problem

The problem we shall address is the following. Suppose we wish to calculate, starting from the "fundamental principles" of QCD, the structure of the deuteron and the scattering of two nucleons at very low energy, with the characteristic momentum probed p much less than the pion mass $m_\pi \approx 140$ MeV. What are the relevant variables? Quarks and gluons are clearly not the *right* variables. We introduce instead the nucleon field ψ in the Lagrangian as a matter field [#3]. Next we have the option of having or not having the pion field figure explicitly in the Lagrangian. In the real world, the pion has a mass of about 140 MeV, much larger than the momentum scale we are interested in but in the chiral limit, it would be degenerate with the vacuum. So one might ask whether it is necessary to retain the pion in the framework even when the pion mass is taken into account. To address this question, we will work to the next-to-leading order (NLO) in the momentum expansion a la Weinberg [12].[#4] It is then sufficient to take the two-nucleon potential written to NLO and insert it into an infinite series of reducible graphs to be summed – which is equivalent to solving the Schrödinger or Lippman-Schwinger equation with that potential.

For the sake of completeness of presentation, we first describe the general features of the counting rules adopted in this article. The way these counting rules are implemented in our formulation will become clear as we describe how the terms that enter into the observables are computed.

For processes whose typical momentum scale can be characterized by the pion mass, the pion field should figure explicitly in the Lagrangian. With nucleons treated as "heavy"

[#2] The latest phase-shift analyses of the two-nucleon data and the construction of phenomenological potentials to reproduce the resulting phase shifts have reached such a level that all the existing data below ∼350 MeV can be reproduced with high precision[17, 11]. We refer to these state-of-the art nucleon-nucleon potentials as high-quality phenomenological potentials. The two-nucleon wave functions generated by these potentials are referred to as high-quality phenomenological wave functions.

[#3] In principle we could start with a Lagrangian consisting of the Goldstone meson fields only, obtain the baryons as solitons ("skyrmions") and then proceed to compute the fluctuations. But this line of research has not yet met with a quantitative success. See [14] for a discussion on this matter.

[#4] Recently, Weinberg's counting rule (2) has been questioned by the authors of [7] in connection with the large scattering length for the S-wave np scattering. Our use of the Weinberg counting is not affected by this criticism. This matter is discussed in Appendix.

and the anti-nucleon degrees of freedom integrated out, one obtains a consistent and systematic expansion scheme – heavy-baryon chiral perturbation theory – where nucleons and pions are the dynamical fields. Let Q be the typical momentum scale of the process or the pion mass, which is regarded as "small" compared to the chiral scale $\Lambda_\chi \sim 1$ GeV. The counting rule given by Weinberg[12] is that, a Feynman graph comprised of A nucleons, N_E external (electroweak) fields, L *irreducible* loops and C-separated pieces is order of $\mathcal{O}(Q^\nu)$ with

$$\text{ChPT:} \quad \nu = 2L + 2(C-1) + 2 - (A + N_E) + \sum_i \bar{\nu}_i, \tag{2}$$

$$\bar{\nu}_i \equiv d_i + \frac{n_i}{2} + e_i - 2 \tag{3}$$

where we have characterized each vertex i by d_i the number of derivatives and/or the power of the pion mass, n_i that of nucleon fields and e_i that of external (electroweak) fields. The quantities $\bar{\nu}_i$ and C are defined so that $\bar{\nu}_i \geq 0$ even in the presence of external fields [15] and $(C-1) = 0$ for connected diagrams.

Now if we limit ourselves to a very low-momentum regime, say, $p \ll m_\pi$, then even the pion field can be integrated out and we are left only with the nucleon field. Since we have integrated out the anti-nucleon degrees of freedom as well as all other meson fields, there are no irreducible loops and the corresponding counting rule can be obtained simply by putting the number of loops L equal to zero,

$$\text{EFT:} \quad \nu = 2(C-1) + 2 - (A + N_E) + \sum_i \bar{\nu}_i \tag{4}$$

with the same definition for $\bar{\nu}_i$ given in eq.(3). But the meaning of d_i in eq.(3) is changed: Since the pion field is integrated out, the d_i stands here only for the number of derivatives, and not the power of the pion mass. Then, up to NLO, the resulting potential consists of contact interactions and their two spatial derivatives.

Electromagnetic interactions have different counting rules and should be treated separately. Since the Coulomb interaction (between protons) is given as $\frac{\alpha}{\mathbf{q}^2}$ (where $\alpha \simeq 1/137$ is the fine-structure constant and $\mathbf{q}$ is the momentum transferred), it is of relevance in an extremely small momentum region while it becomes irrelevant in other regions due to the smallness of α. Since we are interested in the proton fusion at threshold, we will explicitly include the Coulomb potential in the *pp* sector.

If we increase the typical probe momentum to a value close to but lower than the pion mass, the pion degree of freedom will become "marginal" and its explicit presence will improve the convergence of the series. In this article, we will perform an NLO calculation both with and without the pion field, thereby revealing its interplay in the effective field theory (EFT). To this order, pion loops do not enter and hence we can simply focus on the (irreducible) potential. Our *unregulated* potential $\mathcal{V}$ has the form

$$\mathcal{V}(\mathbf{q}) = -\tau_1 \cdot \tau_2 \frac{g_A^2}{4f_\pi^2} \frac{\sigma_1 \cdot \mathbf{q}\, \sigma_2 \cdot \mathbf{q}}{\mathbf{q}^2 + m_\pi^2} + \frac{4\pi}{M}\left[C_0 + (C_2\delta^{ij} + D_2\sigma^{ij})q^i q^j\right] + Z_1 Z_2 \frac{\alpha}{\mathbf{q}^2} \tag{5}$$

with

$$\sigma^{ij} = \frac{3}{\sqrt{8}} \left(\frac{\sigma_1^i \sigma_2^j + \sigma_1^j \sigma_2^i}{2} - \frac{\delta^{ij}}{3} \sigma_1 \cdot \sigma_2 \right), \tag{6}$$

where $\mathbf{q}$ is the momentum transferred, $g_A \simeq 1.26$ the axial-vector coupling constant, $f_\pi \simeq 93$ MeV the pion decay constant, $M \simeq 940$ MeV the nucleon mass, and $Z_1 Z_2 = 1$ for the pp channel and zero otherwise. For the proton fusion process, we will also consider $\mathcal{O}(\alpha^2)$ corrections, i.e., the vacuum polarization (VP) potential and the two-photon-exchange (C2) potential; as in [10], these will be treated perturbatively. The parameters C's and D_2 are defined for each channel. The D_2 term, however, is effective only for the spin-triplet channel. Thus, there are two parameters (C_0 and C_2) for each of the pp and np $^1S_0$ channels, and three (C_0, C_2 and D_2) for the $^3S_1$ channel.

It is perhaps useful to explain how the potential (5) is to be understood. The first (nonlocal) term is the pion exchange involving the Goldstone boson and hence completely known. The (local) terms in the square brackets represent the effect inherited from the degrees of freedom that have been integrated out, for instance the heavy mesons whose masses are much higher than the cutoff. It should be noted that when the pion is also integrated out, its effect will appear in the local terms, modifying the constants. This means that even if we drop the first term of (5), we still have part, if not most, of pionic effects in the theory.

Inserted into an infinite bubble sum required to describe bound and quasi-bound states, the potential (5) will generate an infinite series of divergent terms unless the integrals are suitably regularized. This is expected in an effective theory which is not renormalizable in the conventional (Dyson) sense. As announced we shall regularize the divergences with a momentum cutoff. In [9] where only nucleonic contact terms and their derivatives were taken into consideration, a regularization that preserves the separability of the potential was used. Of course, the precise form of the regulator should not matter [5]. Since both the Coulomb interaction and the one-pion-exchange potential are non-separable, we shall adopt a slightly different form of the regularization: in coordinate space,

$$V(\mathbf{r}) \equiv \int \frac{d^3\mathbf{q}}{(2\pi)^3} \, \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}} \, S_\Lambda(\mathbf{q}^2) \, \mathcal{V}(\mathbf{q}), \tag{7}$$

where $S_\Lambda(\mathbf{q}^2)$ is the regulator with a cutoff Λ. For our purpose it is convenient to take the Gaussian regulator[9, 16, 5],

$$S_\Lambda(\mathbf{q}^2) = \exp\left(-\frac{\mathbf{q}^2}{2\Lambda^2} \right). \tag{8}$$

Inserting eqs.(5, 8) into eq.(7), we obtain

$$\begin{aligned} V(\mathbf{r}) &= \alpha_\pi \frac{\tau_1 \cdot \tau_2}{3} \left[\sigma_1 \cdot \sigma_2 \, v_\Lambda(r, m_\pi) + S_{12}(\hat{r}) \, t_\Lambda(r, m_\pi) \right] \\ &+ \frac{4\pi}{M} \left[C_0' - C_2 \nabla_{\mathbf{r}}^2 - \frac{S_{12}(\hat{r})}{\sqrt{8}} D_2 r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \right] \delta_\Lambda^3(\mathbf{r}) \\ &+ Z_1 Z_2 \alpha \, v_\Lambda(r, 0) \end{aligned} \tag{9}$$

with $S_{12}(\hat{r}) \equiv 3\sigma_1 \cdot \hat{r}\, \sigma_2 \cdot \hat{r} - \sigma_1 \cdot \sigma_2$,

$$\delta^3_\Lambda(\mathbf{r}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}}\, S_\Lambda(\mathbf{q}^2)\,, \tag{10}$$

$$v_\Lambda(r,m) = \frac{4\pi}{S_\Lambda(-m^2)} \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}}\, \frac{S_\Lambda(\mathbf{q})}{\mathbf{q}^2 + m^2}, \tag{11}$$

$$t_\Lambda(r,m) = \frac{r}{m^2}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} v_\Lambda(r,m) \tag{12}$$

and

$$\alpha_\pi \equiv S_\Lambda(-m_\pi^2)\frac{g_A^2 m_\pi^2}{16\pi f_\pi^2}. \tag{13}$$

The factor $S_\Lambda^{-1}(-m_\pi^2)$ in the definition of $v_\Lambda(r,m)$ is introduced so that $v_\Lambda(r,m)$ and $t_\Lambda(r,m)$ become Yukawa functions in the large-r limit:

$$\begin{aligned} \lim_{\Lambda r \gg 1} v_\Lambda(r,m) &= \frac{\mathrm{e}^{-mr}}{r}, \\ \lim_{\Lambda r \gg 1} t_\Lambda(r,m) &= \left(1 + \frac{3}{mr} + \frac{3}{(mr)^2}\right)\frac{\mathrm{e}^{-mr}}{r}. \end{aligned} \tag{14}$$

In eq.(9), we have absorbed the $\delta^3_\Lambda(\mathbf{r})$ term appearing in the one-pion-exchange potential into the nucleon contact interactions by redefining C_0 as

$$C_0' \equiv C_0 - \frac{M\alpha_\pi}{3m_\pi^2 S_\Lambda(-m_\pi^2)}\tau_1 \cdot \tau_2\, \sigma_1 \cdot \sigma_2. \tag{15}$$

Hereafter we will omit the prime on C_0. As for the one-pion-exchange potential, we set $\alpha_\pi = 0.075$ following [11, 17].[#5]

Let us write the potential for each channel of interest.

• **The** np **and** pp $^1S_0$ **channel**: Here tensor interactions are ineffective and so the potential reduces, with eq.(19), to

$$\begin{aligned} V(\mathbf{r}; {}^1S_0) &= -\alpha_\pi\, v_\Lambda(r, m_\pi) + \frac{4\pi}{M}\left[C_0^{1\mathrm{S}0} - \Lambda^2(\Lambda^2 r^2 - 3)\, C_2^{1\mathrm{S}0}\right]\delta^3_\Lambda(\mathbf{r}) \\ &+ Z_1 Z_2 \alpha\, v_\Lambda(r,0). \end{aligned} \tag{16}$$

The coefficients $C_{0,2}^{1\mathrm{S}0}$ differ in the pp channel from those in the np channel due to the charge-independence breaking interactions.

[#5] Note that our EFT potential (5) resembles the potential used in Nijmegen group's multi-energy analysis [17]. They used – in fitting their potential – 1787 pp and 2514 np accurate scattering data below $T_{\mathrm{lab}} = 350$ MeV published between 1955 and 1992. They separated their potential into the long-range potential V_L and the short-range potential V_S such that $V(r) = V_L(r)$ for $r \geq b$, and $V(r) = V_S(r)$ otherwise, with $b = 1.4$ fm. Their long-range potential consists of electromagnetic interactions and one-pion-exchange potentials, as well as heavy-meson-exchange (such as ω, ρ and η) potentials, but since V_L is applied only for $r \geq b$, the heavy-meson exchange is irrelevant except for achieving high precision in the large momentum region. The short-ranged part is parameterized by a square-well potential (of radius b) with an E-dependent depth. The E-dependence is given by a finite series in increasing powers of E. Taking $1 \sim 3$ terms for each channel (except for the pp $^1S_0$ channel for which four terms are retained), they achieve a very accurate fit with $\chi^2/\mathrm{data} = 1.08$.

• **The deuteron $^3S_1 - {}^3D_1$ coupled channel**: The potential is of the form

$$V(\mathbf{r}; {}^3S_1 - {}^3D_1) \;=\; V_C^d(r) + S_{12}(\hat{r})\, V_T^d(r) \tag{17}$$

where, due to eq.(19),

$$\begin{aligned} V_C^d(r) &= -\alpha_\pi\, v_\Lambda(r, m_\pi) + \frac{4\pi}{M}\left[C_0^{3S1} - \Lambda^2(\Lambda^2 r^2 - 3)\, C_2^{3S1}\right] \delta_\Lambda^3(\mathbf{r}), \\ V_T^d(r) &= -\alpha_\pi\, t_\Lambda(r, m_\pi) - \frac{4\pi}{M}\frac{D_2^{3S1}}{\sqrt{8}}\, \Lambda^4 r^2\, \delta_\Lambda^3(\mathbf{r}). \end{aligned} \tag{18}$$

With the regulator (8), it is a simple matter to work out the explicit forms of the quantities we need:

$$\nabla_{\mathbf{r}}^2 \delta_\Lambda^3(\mathbf{r}) = \Lambda^2(\Lambda^2 r^2 - 3)\, \delta_\Lambda^3(\mathbf{r}), \quad r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\delta_\Lambda^3(\mathbf{r}) = \Lambda^4 r^2\, \delta_\Lambda^3(\mathbf{r}) \tag{19}$$

and

$$\delta_\Lambda^3(\mathbf{r}) \;=\; \frac{\Lambda^3 \exp\left[-\frac{\Lambda^2 r^2}{2}\right]}{(2\pi)^{\frac{3}{2}}}, \tag{20}$$

$$v_\Lambda(r, m) \;=\; \frac{1}{2r}\left[\mathrm{e}^{-mr}\,\mathrm{erfc}\left(\frac{-\Lambda r + \frac{m}{\Lambda}}{\sqrt{2}}\right) - (r \to -r)\right], \tag{21}$$

$$v_\Lambda(r, 0) \;=\; \frac{1}{r}\,\mathrm{erf}\left(\frac{\Lambda r}{\sqrt{2}}\right) \tag{22}$$

where $\mathrm{erf}(x)$ and $\mathrm{erfc}(x) \equiv 1 - \mathrm{erf}(x)$ are the error functions,

$$\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x dt\, \mathrm{e}^{-t^2}. \tag{23}$$

Finally the radial functions, $u_0(r)$ for $^1S_0$ and $u_d(r)$ and $w_d(r)$ for $^3S_1 - {}^3D_1$, are obtained by solving the Schrödinger equations

$$u_0''(r) + M\left[E - V(r; {}^1S_0)\right] u_0(r) = 0, \tag{24}$$

and

$$\begin{aligned} u_d''(r) + M\left[E - V_C^d(r)\right] u_d(r) &= \sqrt{8} M V_T^d(r) w_d(r), \\ w_d''(r) - \frac{6}{r^2} w_d(r) + M\left[E - V_C^d(r) + 2V_T^d(r)\right] w_d(r) &= \sqrt{8} M V_T^d(r) u_d(r), \end{aligned} \tag{25}$$

with the boundary condition

$$u_0(0) = u_d(0) = w_d(0) = 0. \tag{26}$$

One caveat is in order here. Solving the Schrödinger equation with the irreducible vertex calculated to NLO is consistent with the Weinberg scheme. As pointed out in [16, 7], however, it is not strictly consistent with the chiral counting order by order since the Gaussian cut-off function brings in all powers of $p/\Lambda \sim Q/\Lambda$ and so does the iteration to

all orders of the irreducible vertex that is involved in the evaluation of reducible diagrams. Nevertheless, we believe that our procedure is safe from uncontrollable errors. First of all, in our treatment, the cutoff has a physical meaning and should not be sent to infinity. Therefore there is no problem of the sort described by Beane *et al.*[16] and by Luke and Manohar [18], which one would encounter if one were to send the cutoff to infinity or if one were to use the *naive* dimensional regularization (DR) in the Weinberg scheme. Furthermore to the order we work in computing the irreducible diagrams, our method is consistent with the power counting. Possible errors reside only in higher-order terms that are being dropped. A similar conclusion was reached by Gegelia [29].

3 Renormalization

For a finite cutoff Λ, only finite quantities enter. However renormalization is required to render the theory consistent (see [6] for details). In particular, certain cutoff-dependent terms have to be absorbed into the constants before determining them from empirical data. We proceed as follows. For a given cutoff and for each channel, we determine the constants C's and D's in (5) by relating them to the low-energy scattering parameters appearing in the effective-range formula.

• **The** $np\ ^1S_0$ **channel**: The effective-range formula for this channel is

$$p\cot\delta_0 = -\frac{1}{a_0} + \frac{1}{2}r_0p^2 + \mathcal{O}(p^4) \tag{27}$$

where $p \equiv \sqrt{ME}$ is the relative momentum and the subscript '0' denotes the $np\ ^1S_0$ channel, i.e., $\delta_0 \equiv {}^1\delta_{np}$ and similarly for the scattering length a_0 and the effective range r_0. The low-energy scattering parameters, a_0 and r_0, can be given by the radial wavefunction $u_0(r)$ at zero energy:

$$a_0 = \left.\frac{ru_0'(r) - u_0(r)}{u_0'(r)}\right|_{r\geq R}, \tag{28}$$

$$r_0 = 2\int_0^\infty dr\ \left[\phi_0^2(r) - u_0^2(r)\right], \tag{29}$$

where R denotes the range of the potential such that V(r)=0 for $r \geq R$, and $\phi_0(r)$ is the large-r asymptotic wavefunction of $u_0(r)$,

$$\lim_{r\to\infty} u_0(r) = \phi_0(r) = 1 - \frac{r}{a_0}. \tag{30}$$

The experimental values of a_0 and r_0 which we shall use to fix C_0^{np} and C_2^{np} are:

$$a_0^{\rm exp} = -23.749(8)\ {\rm fm}, \quad r_0^{\rm exp} = 2.81(5)\ {\rm fm}. \tag{31}$$

• **The** $pp\ ^1S_0$ **channel**: In this channel, the infinite-ranged Coulomb potential must be taken into account. There are some subtleties in doing this because there are many

different definitions of the low-energy scattering parameters depending on how the long-ranged $\mathcal{O}(\alpha^2)$ corrections are treated [19]. One reliable scheme was described in full detail in [10], which we will follow here. We define the phase shift δ^C – and consequently the low-energy scattering parameters a^C and r^C – in the *absence* of the potentials of order $\mathcal{O}(\alpha^2)$ and take into account these higher order EM effects by perturbation theory. The expressions obtained in [10] for the low-energy scattering parameters are

$$a^C = \frac{1}{m_p\alpha} \left. \frac{u'_{\rm C+N}(r)\phi_2(r) - u_{\rm C+N}(r)\phi'_2(r)}{u'_{\rm C+N}(r)\phi_1(r) - u_{\rm C+N}(r)\phi'_1(r)} \right|_{r\geq R}, \tag{32}$$

$$r^C = 2\int_0^\infty dr \left[\phi_C^2(r) - u_{\rm C+N}^2(r)\right] \tag{33}$$

with

$$\phi_C(r) = \phi_1(r) - \frac{\phi_2(r)}{m_p\alpha\, a^C}, \tag{34}$$

where $m_p \simeq M$ is the proton mass, $u_{\rm C+N}(r)$ is the radial wavefunction that approaches the function $\phi_C(r)$ asymptotically, and $\phi_1(r)$ and $\phi_2(r)$ are the radial wavefunctions calculated with only the Coulomb interaction at zero energy,

$$\begin{aligned} \phi_1(r) &= 2\sqrt{m_p\alpha r}K_1(2\sqrt{m_p\alpha r}), \\ \phi_2(r) &= \sqrt{m_p\alpha r}I_1(2\sqrt{m_p\alpha r}). \end{aligned} \tag{35}$$

Here $K_1(z)$ and $I_1(z)$ are the modified Bessel functions. In fixing C_0^{pp} and C_2^{pp}, we shall use the following experimental values (extracted from the Nijmegen multi-energy analysis [17]) of a^C and r^C:

$$a^C_{\rm exp} = -7.8196(26)\ {\rm fm}, \quad r^C_{\rm exp} = 2.790(14)\ {\rm fm}. \tag{36}$$

• **The deuteron $^3S_1 - ^3D_1$ channel**: In this channel, we have the choice of renormalizing the theory either at zero energy or at the deuteron pole position $E = -B_d$, where B_d is the deuteron binding energy. Here we shall adopt the latter. The coefficient D_2^d will be determined by the experimentally well-known deuteron D/S ratio η_d. Near the deuteron pole position, the effective-range formula has the form

$$p\cot\delta_d = -\gamma + \frac{1}{2}\rho_d(p^2+\gamma^2) + \mathcal{O}\left[(p^2+\gamma^2)^2\right] \tag{37}$$

where δ_d is the S-wave eigenphase and $\gamma \equiv \sqrt{MB_d}$ is the deuteron wave number. Thus the quantities B_d, ρ_d and η_d are inputs to determine the three constants C_0^d, C_2^d and D_2^d. Since by construction our potential is energy-independent, the wavefunction normalization factor A_s is not an independent quantity. It is related to the input parameters by

$$A_s^2 = \frac{2\gamma}{(1+\eta_d^2)(1-\gamma\rho_d)}. \tag{38}$$

In our numerical calculation, we shall use the following experimental values for B_d, A_s and η_d:

$$B_d^{\rm exp} = 2.224575(9)\ {\rm MeV}, \quad A_s^{\rm exp} = 0.8846(8)\ {\rm fm}^{\frac{1}{2}}, \quad \eta_d^{\rm exp} = 0.0256(4). \tag{39}$$

From eq.(38) we get $\rho_d = 1.7635(46)$ fm.

4 Results

As stated, we shall use the effective-range parameters to fix the coefficients of (5) for the np and pp $^1S_0$ channels and B_d, A_s and η_d for the deuteron channel. Given solutions for the Schrödinger equation with the nucleon-nucleon potential thus obtained and the electroweak current derived to the leading order, we are in a position to carry out a *parameter-free* calculation of the M1 matrix element for the np capture process and the Gamow-Teller matrix element for the pp fusion process. Furthermore, the deuteron static properties such as the charge radius r_d, the quadrupole moment Q_d and the magnetic moment μ_d can be predicted. The precise value of the cutoff is of course not determined by the theory but the consistency of the scheme requires that the results should be insensitive to its precise value provided that it is picked judiciously: e.g., around the pion mass if the pion is integrated out and around the two-pion mass if the pion is present.

The relevant electroweak matrix elements we shall focus on are [#6]

$$\mathcal{M}_{\rm M1} \equiv \int_0^\infty dr\, u_d(r) u_0(r), \tag{40}$$

$$\mathcal{M}_{\rm GT} \equiv \int_0^\infty dr\, u_d(r) u_{\rm C+N}(r). \tag{41}$$

The $\mathcal{O}(\alpha^2)$ (VP and C2) corrections to the GT matrix element are calculated perturbatively[10] using the potentials

$$\begin{aligned} V_{\rm VP}(r) &= \frac{2\alpha^2}{3\pi}\int_1^\infty dx\, {\rm e}^{-2m_e r x}\left(1+\frac{1}{2x^2}\right)\frac{\sqrt{x^2-1}}{x^2}, \\ V_{\rm C2}^{\rm C}(r)+V_{\rm C2}^{\rm N}(r) &= -\frac{\alpha}{m_p r}\left(\frac{\alpha}{r}+V_N(r)\right), \end{aligned} \tag{42}$$

where m_e is the electron mass, and $V_N(r)$ denotes the potential (16) but without the Coulomb interaction.

4.1 Without the pion

We first discuss the case where the one-pion-exchange term in eq.(5) is absent. The results with the NLO terms are given in Table 1. There the scattering lengths and effective ranges for the np and pp channels and B_d, A_s and η_d for the deuteron channel are used as inputs to determine the constants of (5). We see that the results as a whole are stable against changes in the cutoff as long as the cutoff is taken above the pion mass. The theoretical values are compared with experimental data when available and with the Argonne v_{18} results[#7] otherwise. There is good agreement between the calculated and experimental values.

[#6] We define these matrix elements with all other factors stripped off. In [9] and [10], $\mathcal{M}_{\rm M1}$ and $\mathcal{M}_{\rm GT}$ were denoted by $M_{\rm 1B}$ and $M_{\rm 1B}^{\rm C+N}$, respectively.

[#7] As we discuss below, the results given by the Argonne v_{18} wavefunctions are fit to experiments and hence can be considered as "experiments".

Table 1: The next-to-leading order (NLO) results *without* the pion field. The static properties of the deuteron, the M1 and GT amplitudes and the VP and C2 contributions to the GT amplitude are listed for various choices of the cutoff Λ. The low-energy input parameters are the scattering lengths and effective ranges for the np and pp $^1S_0$ channels, and B_d, A_s and η_d for the deuteron channel. Their experimental values are given by eqs.(31, 36, 39).

Λ (MeV)	140	150	175	200	225	250	Exp.	v_{18}[11]
r_d (fm)	1.973	1.972	1.974	1.978	1.983	1.987	1.966(7)	1.967
Q_d (fm^2)	0.259	0.268	0.287	0.302	0.312	0.319	0.286	0.270
P_D (%)	2.32	2.83	4.34	6.14	8.09	9.90	–	5.76
μ_d	0.867	0.864	0.855	0.845	0.834	0.823	0.8574	0.847
$\mathcal{M}_{\rm M1}$ (fm)	3.995	3.989	3.973	3.955	3.936	3.918	–	3.979
$\mathcal{M}_{\rm GT}$ (fm)	4.887	4.881	4.864	4.846	4.827	4.810	–	4.859
$\delta_{\rm 1B}^{\rm VP}$ (%)	−0.45	−0.45	−0.45	−0.45	−0.45	−0.44	–	−0.45
$\delta_{\rm 1B}^{\rm C2:C}$ (%)	0.03	0.03	0.03	0.03	0.03	0.03	–	0.03
$\delta_{\rm 1B}^{\rm C2:N}$ (%)	−0.19	−0.19	−0.18	−0.15	−0.12	−0.10	–	−0.21

These results incorporate up to NLO terms. Now, what happens when one limits the calculation to the leading order? The answer is that it does not work. It is not difficult to understand why this is so. Doing a leading order (LO) calculation corresponds to ignoring the q^2 term in (5) and, without the q^2 term – and without the pion, there will be no way to reproduce all the deuteron properties as well as the electroweak matrix elements. As shown in Fig. 1 for fixed CM momenta $p \sim m_\pi/2$ and $p \sim m_\pi$, the low-energy np $^1S_0$ phase shifts calculated in LO (denoted by the dot-dot-dashed curve in Fig. 1) depend strongly on the cutoff. This strong cutoff dependence, however, is considerably reduced, when the NLO term is included (dashed curve). The improvement is more dramatic, the lower the momentum probed is. The introduction of the pion field makes the range of the relevant cutoff considerably greater, approaching the next momentum scale involved, namely, $\gtrsim 2m_\pi$. Thus the consistency condition for a viable effective theory with a given set of effective degrees of freedom requires going to sufficiently high order – NLO in the present case. To what order the calculation needs to be pushed and what additional degrees of freedom are required will of course depend on the problem one is addressing.

4.2 With the pion

We shall consider the cases with and without the $\mathcal{O}(q^2)$ term in (5), the former corresponding to LO and the latter to NLO.

• **The LO calculation**: Here the scattering lengths for the $^1S_0$ channel and the deuteron binding energy B_d are inputs. The resulting predictions are given in Table 2.

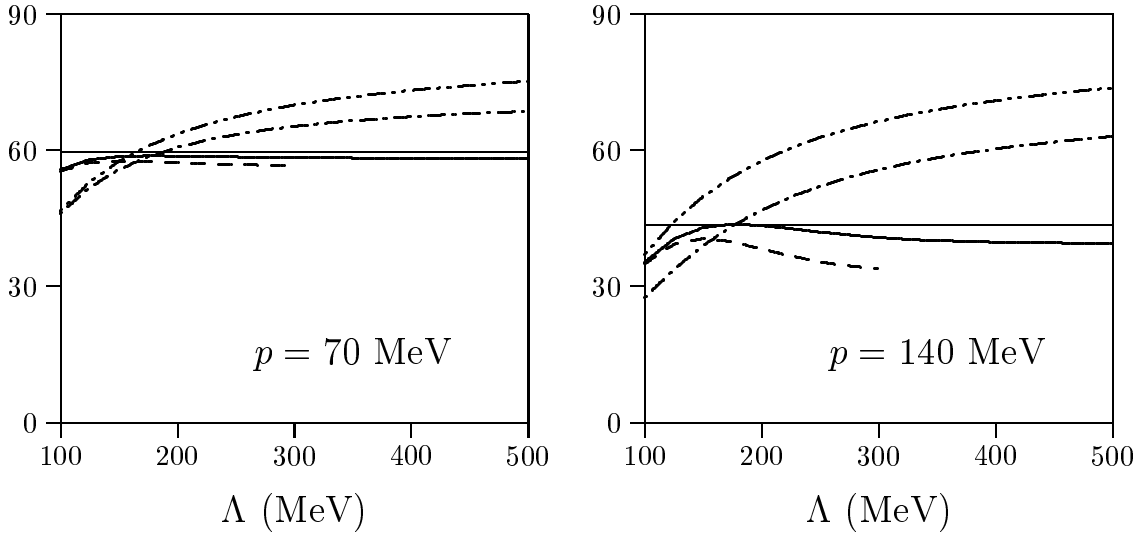

Figure 1: The $np\ ^1S_0$ phase shift (degrees) vs. the cutoff Λ for fixed CM momenta $p = 70$ MeV (left) and $p = 140$ MeV (right). The next-to-leading order (NLO) results are given by the solid (with pions) and dashed (without pions) curves, and the leading-order (LO) results by the dot-dashed (with pion) and dot-dot-dashed (without pion) curves. The horizontal line represents the experimental data obtained from the Nijmegen multi-energy analysis. The NLO result without pions is drawn only up to $\Lambda \gtrsim 300$ MeV, above which the theory becomes meaningless, that is, unnatural as well as inconsistent.

With the pion, the deuteron does exist with its properties qualitatively reproduced. This is of course not surprising to nuclear physicists since the significant role of the pion in the deuteron structure has been known for a long time. What is surprising, however, is that even the electroweak matrix elements are not far from the "experimental values", though some $4 \sim 5$ % higher. The effective ranges are not controlled without the NLO potential, so it is clear that the phase shifts can be reproduced only for very low energies. Actually we observe in Fig. 1 that while the LO result with the pion (denoted by dot-dashed) is better than the LO results without the pion (dot-dot-dashed curve), it is much worse than the NLO result without the pion (dashed curve). This feature, visible also in Fig. 3, will be discussed in more detail later. An important point to note here is that when the pion field is present, the cutoff climbs up to roughly twice the pion mass. This is consistent with our expectation.

• **The NLO calculation**: The pion contribution is fixed by chiral symmetry, so the parameters of the theory are determined in the same way as for the NLO calculation in the absence of the pion. The results are given in Table 3.

The presence of the pion in the NLO calculation brings a noticeable improvement in the accuracy of the prediction making the dependence on the cutoff markedly weaker. This is true not only for the static properties (Table 3) but also for the phase shift for $p \approx m_\pi$ (see the right panel of Fig.1). The cutoff required is roughly twice that of the pionless

Table 2: The leading order (LO) results *with* pions. Note that only the scattering lengths for the $^1S_0$ channel and the deuteron binding energy are inputs. All the quantities except C_0's are predicted; in particular, the wavefunction normalization A_s and the D/S ratio η_d are *not* input but predicted here.

Λ (MeV)	300	400	500	600	700	Exp.	v_{18}[11]
C_0^{np} (fm)	-0.828	-0.621	-0.501	-0.422	-0.366		
C_0^{pp} (fm)	-0.810	-0.615	-0.500	-0.423	-0.367		
C_0^{d} (fm)	-0.979	-0.384	-0.144	0.795	0.789		
A_s ($\mathrm{fm}^{-\frac{1}{2}}$)	0.853	0.854	0.861	0.866	0.869	0.8846(8)	0.885
η_d	0.0227	0.0250	0.0260	0.0263	0.0265	0.0256(4)	0.0250
r_d (fm)	1.897	1.902	1.918	1.930	1.936	1.966(7)	1.967
Q_d (fm^2)	0.218	0.249	0.266	0.274	0.277	0.286	0.270
P_D (%)	2.65	4.59	5.97	6.73	7.08	–	5.76
μ_d	0.865	0.854	0.846	0.841	0.839	0.8574	0.847
$\mathcal{M}_{\rm M1}$ (fm)	4.176	4.186	4.164	4.140	4.122	–	3.979
$\mathcal{M}_{\rm GT}$ (fm)	5.054	5.063	5.042	5.019	5.002	–	4.859

case and this again is consistent with our expectation. Note that the charge radius, the quadrupole moment and the magnetic moment of the deuteron all come out very close to the experimental values, indicating that exchange-current corrections, if any, should be tiny. As remarked in the discussion section, this is expected on a general ground. The M1 matrix element and the GT matrix element receive a few per cents exchange-current corrections which are well controlled for the former and somewhat less well controlled for the latter. See later on this matter.

To give a more quantitative idea as to how much the incorporation of the pionic degree of freedom reduces the cut-off dependence of our calculation, it is informative to consider $\mathcal{E}(\mathrm{M1})$ and $\mathcal{E}(\mathrm{GT})$ defined by

$$\mathcal{E}(\mathrm{M1}) \equiv \frac{\mathcal{M}_{\rm M1}^{\rm th} - \mathcal{M}_{\rm M1}^{v18}}{\mathcal{M}_{\rm M1}^{v18}}, \quad \mathcal{E}(\mathrm{GT}) \equiv \frac{\mathcal{M}_{\rm GT}^{\rm th} - \mathcal{M}_{\rm GT}^{v18}}{\mathcal{M}_{\rm GT}^{v18}}, \tag{43}$$

where $\mathcal{M}_{\rm M1}^{\rm th}$ and $\mathcal{M}_{\rm M1}^{v18}$ denote, respectively, the M1 transition matrix element of our NLO calculation and that of the Argonne potential, and similarly for $\mathcal{E}(\mathrm{GT})$. Here we are taking the Argonne potential as "experiment" (see the next subsection for more explanation on this point). In Fig. 2 are plotted the deviations from the "experiment" as a function of the cutoff. We see that as in the case of the phase shift for $p \sim m_\pi$, the inclusion of the pion degree of freedom markedly reduces the cutoff dependence, in conformity with the requirement for a viable effective theory that the more accurate the power expansion, the less cutoff-dependent physical observables should become.

Table 3: The NLO results *with* pion field. See the caption of Table 1.

Λ (MeV)	200	250	300	350	400	500	Exp.	v_{18}[11]
r_d (fm)	1.963	1.965	1.966	1.967	1.968	1.968	1.966(7)	1.967
Q_d (fm^2)	0.261	0.268	0.272	0.273	0.274	0.274	0.286	0.270
P_D (%)	3.16	4.11	4.77	5.16	5.35	5.39	–	5.76
μ_d	0.862	0.856	0.853	0.850	0.849	0.849	0.857	0.847
$\mathcal{M}_{\rm M1}$ (fm)	3.987	3.976	3.968	3.963	3.958	3.952	–	3.979
$\mathcal{M}_{\rm GT}$ (fm)	4.884	4.874	4.867	4.862	4.859	4.854	–	4.859
$\delta_{\rm 1B}^{\rm VP}$ (%)	−0.46	−0.45	−0.45	−0.45	−0.45	−0.45	–	−0.45
$\delta_{\rm 1B}^{\rm C2:C}$ (%)	0.04	0.03	0.03	0.03	0.03	0.03	–	0.03
$\delta_{\rm 1B}^{\rm C2:N}$ (%)	−0.24	−0.20	−0.15	−0.14	−0.21	−0.42	–	−0.21

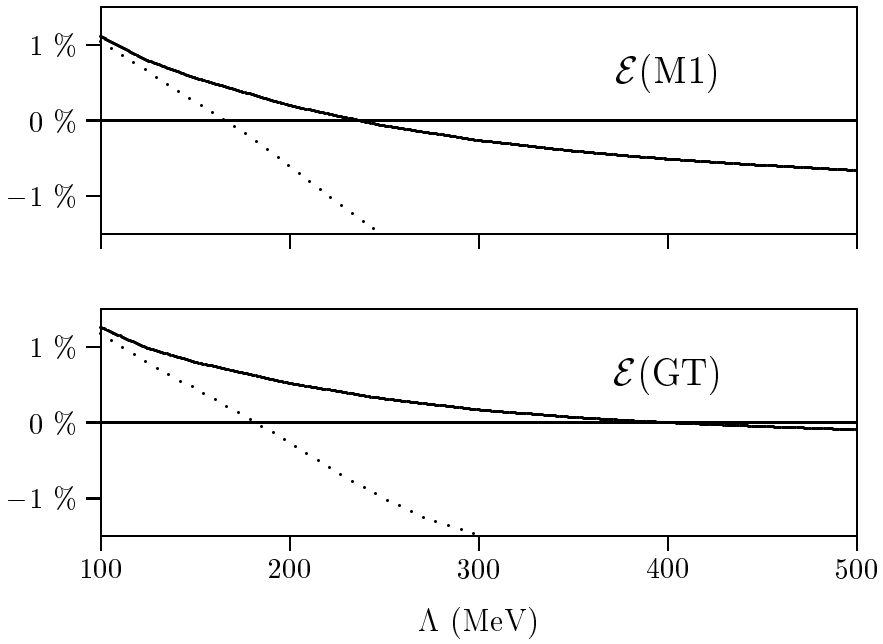

Figure 2: $\mathcal{E}$(M1) (upper) and $\mathcal{E}$(GT) (lower) vs. the cutoff Λ. The solid curves represents the NLO results with pions and the dotted curves without pions.

4.3 Comparison with a realistic phenomenological potential

In our earlier calculations of the *np* capture process [8] and the proton fusion process [10], we resorted to the wavefunctions obtained from the Argonne v_{18} potential – which is one of the most realistic phenomenological potentials fit to *all* available two-nucleon data up to $T_{\rm Lab}$=350 MeV – to calculate the matrix elements of the M1 and GT operators derived to the next-to-next-to leading order (NNLO) in chiral expansion. It might appear to some purists that this "hybrid" approach bringing in information from outside of the chiral perturbation theory framework is not consistent with the premise of an effective theory and hence throws doubt on the value and validity of such calculations. In this subsection, we argue that such a doubt can be dispelled because the hybrid model turns out to be in precise numerical agreement with the effective theory scheme. For our purpose, it suffices to limit our consideration to the leading-order matrix elements.

In [9], we showed that the leading-order M1 matrix element calculated in the hybrid method was is in good agreement with the result of the effective field theory approach in the absence of the pion. Here we confirm that the M1 matrix element as well as the GT matrix element (figuring in the *pp* fusion process) calculated *both with and without* pions quantitatively support the equivalence of the hybrid approach and the effective field theory approach with cutoff. We see from the NLO calculations given in Tables 1 and 3 that the electroweak matrix elements calculated in NLO in the effective field theory agree with those computed with the Argonne v_{18} within 1 % error, with very little dependence on the cutoff. With the pion included the agreement is even more striking as one can see in Figure 2.

Comparison is made also for the $\mathcal{O}(\alpha^2)$ EM corrections to the *pp* fusion matrix element. These are obviously small corrections, smaller than higher-order power corrections ignored in the calculation but they cannot be ignored in the solar proton burning process as discussed in [20]. As one can see in Tables 1 and 3, the agreement is again almost perfect.

5 Meson-Exchange Currents and Short-Range Correlation

So far we have considered the matrix elements of only one-body electroweak currents. In considering meson-exchange currents that appear as higher-order corrections, it is useful to recall the "chiral filter mechanism" first studied in [15, 21, 22]. The chiral filter mechanism is the statement that, whenever soft-pion exchange is *not forbidden* by symmetry or *kinematically unsuppressed*, the dominant term is given by chiral symmetry and the corrections to it should be suppressed. A corollary to this is that if soft-pion exchange is *forbidden* by symmetry or *kinematically suppressed*, then higher-order terms enter non-negligibly so that the predictive power of the standard chiral perturbation strategy is appreciably weakened if not lost.

According to this argument, we can make the following qualitative statements:

- The corrections to the single-particle M1 matrix element in the np capture process[#8] are given by one-soft-pion exchange (NLO) and loop corrections (NNLO). Since the LO one-body operator comes from the vertex with $\bar{\nu}_i = 1$, these corrections are, respectively, $\mathcal{O}(Q)$ and $\mathcal{O}(Q^3)$ relative to the LO operator. Working to $\mathcal{O}(Q^3)$ relative to the (non-vanishing) leading order, we need to have wavefunctions (or potentials) accurate up to $\mathcal{O}(Q^3)$ for the one-body matrix element, to $\mathcal{O}(Q^2)$ for the soft-pion exchange and to $\mathcal{O}(Q^0)$ for one-loop correction. Since parity allows only even powers in the potential, it is sufficient therefore to have an $\mathcal{O}(Q^2)$ accuracy. Our effective field theory to NLO meets this requirement. Let $\mathcal{M}_{\rm M1}^{\rm total} = \mathcal{M}_{\rm M1}(1+\mathcal{R})$ be the matrix element of the full M1 operator, with $\mathcal{R}$ being the ratio of the matrix element of the exchange current operator to that of the leading M1 operator, $\mathcal{M}_{\rm M1}$. With a "low-quality" phenomenological potential, we will very likely get a poor estimate of the leading one-body matrix element $\mathcal{M}_{\rm M1}$ mostly because of its inability to reproduce the low-energy scattering parameters. Even so, as explained above, the ratio $\mathcal{R}$ calculated with the same wavefunction is expected to be much less sensitive to the quality of the phenomenological potentials. That is, $\mathcal{R}$ should be to a large extent model-independent. Another way of arriving at this conclusion is as follows. As we have shown in this article, the one-body matrix element $\mathcal{M}_{\rm M1}$ can be calculated with great accuracy with the NLO EFT potential as well as with a "high-quality" phenomenological potential such as the Argonne v_{18}, both of which reproduce all the relevant low-energy scattering observables with high precision. Since the total matrix element $\mathcal{M}_{\rm M1}^{\rm total}$ should be the same regardless of whether we adopt the EFT framework developed here or the Argonne v_{18} wavefunctions, we conclude that the ratio $\mathcal{R}$ should not depend on which of the two methods is used.

  One non-trivial consequence of this argument is that the ratio $\mathcal{R}$ should not depend upon how the current is regularized. Since the NLO correction term is made of tree diagrams only, the ratio should be reliably given by the Argonne v_{18} wavefunction. As for the NNLO term, it was found in [8] that all but one term (hereafter referred to as κ term) is fixed by experiments. This κ term which is left undetermined by experiment arises from a NNLO two-body counter term in the current. If one were to use the hybrid method of [8], this κ term – which is of zero range – could not contribute since it is suppressed by the short-range correlation in the wavefunction. Furthermore all zero-range operators that would appear as higher-order counter terms would also be suppressed. On the other hand, if one were to *naively* calculate the matrix element of the same zero-range operators in an EFT with the cut-off regularization used in this paper, the contribution would be non-zero unless the constants (e.g., κ) happen to be all zero. So it would seem that the two approaches give different results for

[#8] The same discussion applies to the time component of the axial current, that is, the axial-charge transitions (first-forbidden β-decays) in nuclei. For detailed discussions, see [22].

non-zero κ's at NNLO and higher orders. We conjecture that this difference can be at least partially resolved by appealing to the operator product expansion (OPE) in the wavefunction discussed in the context of EFT by Lepage [5]. The OPE implements in the wavefunction the physics of perturbative QCD which is left out in the EFT framework discussed here. For long-wavelength operators (such as the single-particle M1 and GT operators), the effect of OPE would not be important. However, operators probing short-distance physics such as the zero-range κ term should be affected by the OPE factor in the wavefunction. Our conjecture is that the short-range correlation in the standard nuclear physics approach and the OPE in the EFT approach a la Lepage capture qualitatively the same physics. At present we do not know how to make the connection between the two descriptions. We suggest that the lack of knowledge on the precise connection accounts for the uncertainty in the former approach manifested in different results for different forms of short-range correlation functions and for different short-range correlation sizes. This uncertainty – small in some cases like the np capture – is the price to pay in "killing" the higher-order counter terms, which we expect to be unimportant for very low-energy processes we have been considering but significant for higher-energy probes.

For completeness we mention that an NNLO calculation for the exchange current contribution performed using the Argonne v_{18} wavefunctions gave $\sim 10\%$ contribution to the np capture cross section leading to a precise agreement with the experiment [8]. The major contribution was given by the dominant soft-pion term, with only about 1 % coming from NNLO loop corrections. The uncertainty in the calculation was about 1 % in the cross section, representing the above-mentioned uncertainty in the short-range correlation that was used to "kill" the undetermined delta-function counter term.

- The power counting of the GT matrix element is slightly different from that of the M1 operator. Here the one-body GT operator is not suppressed by symmetry but one-soft-pion-exchange current is. As a consequence, the leading correction (one-pion-exchange) is of order of $\mathcal{O}(Q^3)$ while loops enter at $\mathcal{O}(Q^4)$ or higher compared to the one-body operator. This means that the tree-order contributions involving one pion-exchange two-body operators and loop contributions and/or higher-dimensional counter terms that come at next order must be comparable in importance. So far a consistent calculation exists only in tree order, which predicts a $\sim 4\%$ correction (an enhancement) to the leading GT matrix element squared[10]. The (corollary to the) chiral filter mechanism implies that this correction could receive a substantial contribution from higher-order or short-distance corrections. We should mention that the Landau-Migdal correction to g_A in medium discovered a long time ago[23][#9] corresponds to this class of effects.

[#9] A more modern interpretation in heavy-baryon chiral perturbation theory is given in [24].

- The exchange-current corrections to the deuteron properties, e.g., the charge radius, the quadrupole moment and the magnetic moment, are also suppressed by the chiral filter [15, 21]. An additional suppression occurs due to isospin symmetry. This explains why in Tables 1 and 3 the theory without exchange current corrections agrees – almost precisely – with experiments without any further corrections.

6 Breakdown of Effective Field Theory?

Given a cutoff and a set of degrees of freedom chosen to work with, where is the given theory expected to break down and become inaccurate? By way of answering this question, let us look at low-energy $np\ ^1S_0$ phase-shifts. In Fig. 3 are plotted the phase shifts vs. the CM momentum p for the lower and upper limits of the cutoff both in the absence and in the presence of the pion degree of freedom. The range $\Lambda = (100 \sim 300)$ MeV represents the range of the cutoff in which the NLO theory without pions should be valid and the range $\Lambda = (200 \sim 500)$ MeV is the corresponding range when the pion field is incorporated.

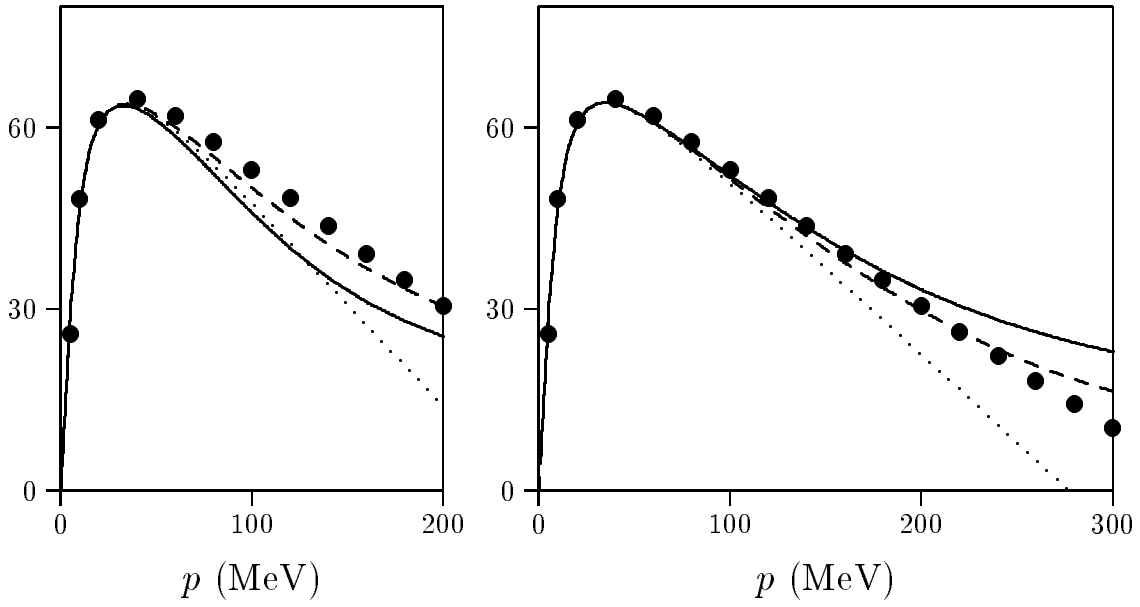

Figure 3: The $np\ ^1S_0$ phase shift (degrees) vs. the center-of-mass (CM) momentum p. We show the predicted results both without (left) and with (right) the pion field. For each case, our NLO results are given for two extreme values of the cutoff: The solid curve represents the lower limit, $\Lambda = 100$ MeV (without the pion) and 200 MeV (with the pion), and the dotted curve the upper limit, $\Lambda = 300$ MeV (without the pion) and 500 MeV (with the pion). The "experimental points" obtained from the Nijmegen multi-energy analysis are given by the solid circles. Our theory with $\Lambda = 150$ MeV (without the pion) and $\Lambda = 250$ (with the pion) are shown by the dashed line to show that the theory is in almost perfect agreement with the data up to $p \lesssim 200$ MeV.

There are several lessons one can learn from Fig. 3. By fiat, all the constants of the theory are fixed by the scattering length and the effective range, that is, the first two

terms in the effective-range expansion are fixed. However the Gaussian cutoff introduces terms of $\mathcal{O}[(p/\Lambda)^n]$ for $n \geq 3$. The theory works fairly independently of the cutoff up to $p \sim 100$ MeV if the pion field is included, and up to $p \sim 50$ MeV if the pion is absent; beyond these limits the cutoff dependence becomes noticeable for both cases. Since the two-term effective-range formula fits data up to $p \sim 140$ MeV,[#10] higher-order terms must be responsible for the *deviation* from the experiment. Thus, in order to recover the fit, terms of order $\mathcal{O}[(p/\Lambda)^n]$ with $n \geq 3$ must be included in the (irreducible) potential V. In fact the two-term effective-range formula works so well due to the fact that the third term of the effective-range formula, effective volume term, is very close to zero[25]. It then means that the two-term effective-range formula is essentially the same with the three-term effective-range formula with the effective volume term fine-tuned to zero. In order for this to happen in our theory, the contribution from the Gaussian cutoff and the corresponding higher-order counter term – which must be added – should cancel to give a vanishingly small volume term. Therefore it would be "unnatural" if our NLO calculation, without involving a "fine-tuning", were to work as well as the "three-term" effective-range formula. The left panel of Fig. 3 (NLO without the pion field) is thus "natural". On the other hand, the right panel of Fig. 3 (NLO with the pion field) has the accuracy comparable to the "three-term" effective-range formula, which should be understood as the role of the pion field.

Given that we are truncating the series, thereby making the theory manifestly approximate, it seems reasonable to exploit the insensitivity to the precise value of the cutoff and do a "best fit." For this, we shall pick – within the range allowed – a specific value for cutoff in fitting experimental data. We should stress that this has nothing to do with a "fine-tuning" usually associated with an unnatural theory. From Table 3 we can see that, when the pion field is present, $\Lambda \simeq 250$ MeV gives an overall fit of the deuteron properties as well as electroweak matrix elements. In Fig. 3 is plotted the phase shift prediction for this cutoff. The agreement with experiments is excellent up to $p \lesssim 200$ MeV.[#11] Thus for this value of the cutoff, *all* low-energy two-nucleon properties are accurately described by the theory.

We can do a similar "best-fitting" in the absence of the pion field and get a result which, though not as good as in the presence of the pion field, is still quite satisfactory as one can see in the left panel of Fig. 3 for $\Lambda \simeq 150$ MeV.

[#10] This accuracy of the effective-range formula to $p \sim 140$ MeV is somewhat unnatural from the point of view of EFT. It is perhaps fortuitous. Intuitively one would expect that the effective-range formula should work only up to $p \sim m_\pi/2$, since the S-matrix element contains the left-hand cut starting at $p^2 = -(m_\pi/2)^2$.

[#11] In [7], an even more impressive agreement for the phase shift was obtained with the pion field included. However it should be noted that to achieve a better global fitting, the authors used values of the coefficients of the potential that are different from the ones determined by their renormalization procedure. As a consequence, the description for the threshold region became less reliable.

7 Discussion

We have shown in this paper that the idea of effective field theory in its simplest form works remarkably well in nuclear physics of two-nucleon systems at low energy. We have used the cut-off regularization which renders the notion of effective theory physically transparent. It has been seen that the cut-off regularization also renders moot [4] the issue of whether there is a breakdown in Weinberg's counting rule, which consists in applying the chiral counting only to irreducible diagrams, and subsequently summing up an infinite series of reducible graphs containing irreducible vertices of a specified order.

We have further shown that the "hybrid approach" used in [8] to calculate the $n+p \to d+\gamma$ cross section in the framework of chiral perturbation theory (in the Weinberg version) is fully justified by a systematic effective field theory technique with a finite cutoff.

One of the important by-products of the effective field theory for nuclei is precise calculations of nuclear weak processes that figure importantly in astrophysics. In [10], we computed in the "hybrid approach" the cross sections for the solar proton burning process $p+p \to d+e^+ +\nu_e$ to an accuracy of 1 %,[#12] and this has provided strong support for the estimate used by Bahcall and collaborators for the solar neutrino problem[20, 26, 27]. In particular it has resolved the possible controversy as to whether or not a field theoretic approach would give a result basically different from the classic nuclear physics approach which is based on Schrödinger equation with realistic phenomenological N-N interactions[20]. Another potentially important by-product is that with the pion field incorporated, we should be able to treat meson-exchange currents to one-loop order (that is, $\mathcal{O}(Q^4)$ relative to the leading GT operator). This would allow a model-independent calculation of processes like[#13] [28]

$$^3\mathrm{He} + p \to {}^4\mathrm{He} + e^+ + \nu_e \tag{44}$$

which may be measured at the SuperKamiokande and SNO.

As often heralded by the practitioners of the dimensional regularization (DR), one of the advantages of DR over other regularizations is that the DR scheme preserves chiral invariance and Lorentz invariance. Indeed, while giving an excellent result in a non-relativistic regime, the cut-off regularization as used here loses manifest chiral invariance and Lorentz invariance. This might make higher-order loop calculations difficult, if not impossible. Fortunately, in the tree-order treatment of meson exchange currents for the electroweak processes as well as the other two-nucleon processes considered in this paper, we did not have to deal with chirally non-invariant counter terms. However, this problem does show up at the level of loop calculations. We have not investigated this problem in detail but

[#12] The main caveat in [10] is in the meson-exchange contribution which comes out to be about 4 % when calculated to the chiral order $\mathcal{O}(Q^3)$. The higher order contribution starts from $\mathcal{O}(Q^4)$, and may not be necessarily negligible compared with the $\mathcal{O}(Q^3)$ contribution. So a more "conservative" error estimate may be 4 %, instead of 1 %.

[#13] We would like to thank John Bahcall for calling our attention to this problem.

in calculating the process in (44), where meson-exchange current corrections are expected to be more important than in the cases we have considered so far, this issue will have to be resolved. A related issue was investigated by the authors of [6], who suggest that the difficulty is not insurmountable. Their work indicates a promising way of dealing with loop corrections in a finite cut-off field theory for the process (44) as well as for the proton fusion process.

Acknowledgments

We are grateful for correspondence from Gerry Brown and John Bahcall whose suggestions triggered part of this work. One of us (TSP) is indebted to Professor Francisco Fernández for the hospitality at Grupo de Física Nuclear, Universidad de Salamanca, where most of this work was done and to Professor C.W. Kim for his hospitality at Korea Institute for Advanced Study where this paper was finalized. The work of KK was partially supported by the NSF Granty No. PHYS-9602000 and that of DPM by the KOSEF through CTP of SNU and by the Korea Ministry of Education.

Appendix

We briefly comment on a recent discussion [7] on Weinberg's counting rule. For related discussions on this issue, see [4, 29, 30, 31, 32, 33]. More recently, Birse, McGovern and Richardson [34] pointed out that for two-nucleon scattering with a quasi-bound state nearby as involved in these discussions, fluctuations should be made around a non-trivial fixed point. We shall not address this issue here.

The authors of [7] have argued that the counting rule eq.(2) breaks down when applied to *reducible* graphs, which need to be summed to all orders to describe bound or quasi-bound states. If one were to use the usual dimensional regularization, this infinite series with a finite set of irreducible graphs for a potential would require an infinite number of counter terms to regularize the series [18].

In constructing counting rules of low-energy effective theories that are based on derivative expansion, subtlety (if any) resides in loop integrals. To be definite, we consider an n-dimensional integral of a function $f(\mathbf{p}, \mathbf{q})$, where $\mathbf{p}$ stands for an external momentum and $\mathbf{q}$ a loop-momentum,

$$I^\Lambda(\mathbf{p}) \equiv \int^\Lambda d^n\mathbf{q}\, f(\mathbf{p}, \mathbf{q}), \tag{45}$$

where the superscript Λ indicates that the quantity is regulated with a cutoff Λ. Let $f(\mathbf{p}, \mathbf{q})$ be of order of Q^{ν_f} when both $\mathbf{p}$ and $\mathbf{q}$ are of order of Q. Weinberg assumed that the integral should be counted as

$$I^\Lambda(\mathbf{p}) = \mathcal{O}(Q^{n+\nu_f}). \tag{46}$$

Note that we do not write down here explicitly the power of the "large scale" Λ. If one naively assumes that $I^\Lambda(\mathbf{p}) \sim \Lambda^n Q^{\nu_f} \sim Q^{\nu_f}$, where $\sim$ stands for "the same order", then one would find that Feynman graphs with arbitrary numbers of loops are not suppressed, which would make the effective theory completely useless. To elucidate Weinberg's assumption (46), let us expand $I^\Lambda(\mathbf{p})$ in decreasing powers of Λ,

$$I^\Lambda(\mathbf{p}) = \sum_{k=0}^{\infty} \Lambda^{n-k} I_k^\Lambda(\mathbf{p}) \tag{47}$$

with

$$I_k^\Lambda(\mathbf{p}) \sim Q^{\nu_f+k}. \tag{48}$$

The first n-terms ($0 \le k < n$) – which diverges when the cutoff Λ goes to infinity – appear as (finite) polynomial in $\mathbf{p}$. If the loop integral is for an irreducible loop, then the polynomials are to be absorbed into the counter terms of the corresponding vertex functions in the renormalization procedure. Then these terms become "invisible" and hence do not affect any physical observables. This implies that, in constructing counting rules, one can effectively count the cutoff of the loop as order of Q, even though the "actual cutoff" Λ is much bigger than Q and is not directly related to Q. In our understanding, this feature

leads to Weinberg's prescription: Assign the "effective cutoff", Λ_*,

$$\Lambda_* \sim Q \ll \Lambda, \tag{49}$$

to any loop integrals in *irreducible* diagrams. It is then to be expected that Weinberg's assumption (46) cannot be simply extended to reducible loops. In fact, the assumption should not work for reducible loops if a system is to have a (quasi) bound state (see below). Another problem is that, for reducible loops, it is not obvious how to absorb the polynomials, $I_k(\mathbf{p})$ with $k < n$, into counter terms. The standard quantum field theory prescription tells us only how to renormalize vertex functions which receive contributions only from irreducible loops and counter terms.

To further expound these difficulties, we examine the counting rules for *reducible* graphs. The Lippman-Schwinger equation is schematically of the form, $T = V + VG^0T$, where V is a *regulated* potential, G^0 the free two-nucleon propagator and T the T-matrix. For a bound state, there is a pole in the T matrix satisfying the homogeneous equation $T = VG^0T$, which implies $VG^0 = 1$. For quasi-bound states, we have $VG^0 = \mathcal{O}(1)$. For simplicity, let us retain only the C_0 (leading-order contact) term and assume the separability of the potential. Then the required condition for a reducible loop graph is

$$C_0 \int^\Lambda d^3\mathbf{q}\, \frac{q^2}{ME - q^2 + i0^+} \sim -C_0 \int^\Lambda dq \sim 1\,. \tag{50}$$

In finite cut-off effective theory, as noted in [9], C_0 scales as Λ^{-1} and the integral $\int^\Lambda dq \sim \Lambda$, so the condition (50) is trivially satisfied. In the Weinberg counting rule (2), as we have surveyed, every loop in n-dimensional space is counted as $\mathcal{O}(Q^n)$. When reducible graphs are present, we need to introduce a modification to this rule. We note that the following rule is consistent with eq.(50): Any *reducible* loop in n-dimensional space should be counted as $\mathcal{O}(Q^{n-1})$. It is also interesting to find that eq.(50) implies that the very natural scale of $|C_0| \sim \Lambda^{-1}$ forces *any* low-energy two-nucleon interaction to become non-perturbative provided that $C_0 < 0$, which is consistent with the shallow-binding of the deuteron and the almost-binding of np $^1S_0$ channel in nature.

In the dimensional regularization (DR) scheme, however, realizing the condition (50) is non-trivial due to the *lemma* $\int d^n\mathbf{q} = 0$. This could be a reason why EFTs based on the naive DR fails in describing low-energy $^1S_0$ scattering [7, 18]. In [7], a new counting rule was suggested involving subtractions not only at $D = 4$ but for all $D \leq 4$, where D is the space-time dimension. In this scheme, the condition corresponding to (50) takes the form

$$C_0\mu \sim 1 \tag{51}$$

with

$$C_0 = \frac{1}{1/a - \mu}, \tag{52}$$

where μ is the renormalization scale. Eq.(52) satisfies the corresponding consistency condition eq.(51), which is responsible for the success of the PDS scheme in contrast to the failure

of the naive dimmensional regularization.[#14] However, if we were to count the scale μ as of order of Q as was done in [7], there would appear an inconsistency with the Georgi-Manohar counting rule [35] that $\frac{4\pi}{M}C_0 \sim \frac{1}{f_\pi^2} \sim \frac{1}{\Lambda^2}$. On the other hand, if one were to adopt the PDS scheme but count the renormalization scale as $\mu \sim \Lambda$ appropriate for *local* operators, then one would be led to $C_0 \sim \frac{1}{\Lambda}$ and hence to Weinberg's counting rule.

[#14] One of the powers of the PDS scheme is the ability to write an analytic expression for C_0 in terms of the *unnaturally* large scattering length a, eq.(52), reflecting a large anomalous dimensiion of the four-Fermi (baryon) interaction. This feature is somewhat buried in the cut-off method although it is implicit in it.

References

[1] See, e.g., S. Weinberg, *The Quantum Theory of Fields II* (Cambridge Press, 1996); Nature **386** (1997) 234.

[2] J. Polchinski, in *Recent Directions in Particle Theory* eds. by J.Harvey and J. Polchinski (World Scientific, Singapore, 1994); R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).

[3] G.P. Lepage, "What is renormalization?", TASI-89 Summer School Lecture.

[4] J. Gegelia, "Are pions perturbative in effective field theory?", nucl-th/9806028.

[5] G.P. Lepage, "How to Renormalize the Schrödinger Equation", nucl-th/9706029.

[6] J.F. Donoghue and B.R. Holstein, hep-ph/9803312; J.F. Donoghue, B.R. Holstein and B. Borasoy, hep-ph/9804281.

[7] D.B. Kaplan, M.J. Savage and M.B. Wise, Phys. Lett. **B 424** (1998) 390; Nucl. Phys. **B 534** (1998) 329; nucl-th/9804032.

[8] T.-S. Park, D.-P. Min and M. Rho, Phys. Rev. Lett. **74** (1995) 4153; Nucl. Phys. **A 596** (1996) 515.

[9] T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, Phys. Rev. **C 58** (1998) R637.

[10] T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, Astrophys. J. **507** (1998) 443.

[11] R.B. Wiringa, V.G.J. Stoks and R. Schiavilla, Phys. Rev. **C 51** (1995) 38 and references therein.

[12] S. Weinberg, Phys. Lett. **B 251** (1990) 288; Nucl. Phys. **B 363** (1991) 3; Phys. Lett. **B 295** (1992) 114.

[13] C. Ordonez, L. Ray and U. van Kolck, Phys. Rev. Lett. **72** (1994) 1982; Phys. Rev. **C 53** (1996) 2086.

[14] M.A. Nowak, M. Rho and I. Zahed, *Chiral Nuclear Dynamics* (World Scientific, Singapore, 1996).

[15] M. Rho, Phys. Rev. Lett. **66** (1991) 1275.

[16] S.R. Beane, T.D. Cohen and D.R. Phillips, nucl-th/9709062 and references therein.

[17] V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester and J.J. de Swart, Phys. Rev. **C 48** (1993) 792.

[18] M. Luke and A.V. Manohar, Phys. Rev. **D 55** (1997) 4129.

[19] J.R. Bergervoet, P.C. van Campen, W.A. van der Sanden, and J.J de Swart, Phys. Rev. **C 38** (1988) 15.

[20] M. Kamionkowski and J.N. Bahcall, Astrophys. J. **420** (1994) 884.

[21] K. Kubodera, J. Delorme and M. Rho, Phys. Rev. Lett. **40** (1978) 755.

[22] T-S. Park, D.-P. Min and M . Rho, Phys. Repts. **233** (1993) 341.

[23] M. Rho, Nucl. Phys. **A 231** (1974) 493; K. Ohta and M. Wakamatsu, Nucl. Phys. **A 234** (1974) 445.

[24] T.-S. Park, H. Jung and D.-P. Min, Phys. Lett. **B 409** (1997) 26.

[25] H. P. Noyes, Phys. Rev. **130** (1963) 2025; C.W.Wong , Nucl. Phys. **A 536** (1992) 269.

[26] J.N. Bahcall, in *Unsolved problems in astrophysics*, eds. by J.N. Bahcall and J.P. Ostriker (Princeton University Press, Princeton, 1997).

[27] For a recent review, see E.G. Adelberger et al, Rev. Mod. Phys. **70** (1998) 1265.

[28] C. Werntz and J.G. Brennan, Phys. Rev. **C 8** (1973) 1545; P.E. Tegner and C. Bargholtz, Astrophys. J. 272 (1983) 311; J. Carlson, D.O. Riska, R. Schiavilla and R.B. Wiringa, Phys. Rev. **C 44** (1991) 619.

[29] J. Gegelia, nucl-th/9805008; nucl-th/9802038; Phys. Lett. **B429** (1998) 227.

[30] J.V. Steele and R.J. Furnstahl, Nucl. Phys. **A 637** (1998) 46.

[31] M.C. Birse, "Renormalising NN scattering: is power counting powerless?", nucl-th/9804028.

[32] S.R. Beane, "Status of Effective Field Theory of NN Scattering", nucl-th/9806070.

[33] D.R. Phillips, "Regularization and the potential of effective field theory in nucleon-nucleon scattering", nucl-th/9804040.

[34] M.C. Birse, J.A. McGovern and K.G. Richardson, "A renormalization-group treatment of two-body scattering", hep-ph/9807302.

[35] A. Manohar and H. Georgi, Nucl. Phys. **B 234**, (1984) 189.